Solitonic Brane World with Completely Localized (Super)Gravity

Alberto Iglesias[*] and Zurab Kakushadze[†]

C.N. Yang Institute for Theoretical Physics
State University of New York, Stony Brook, NY 11794
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Abstract

We construct a solitonic 3-brane solution in the 5-dimensional Einstein-Hilbert-Gauss-Bonnet theory. This solitonic brane is δ-function like, and has the property that gravity is completely localized on the brane. That is, there are no propagating degrees of freedom in the bulk, while on the brane we have purely 4-dimensional Einstein gravity. Thus, albeit the classical background is 5-dimensional, the quantum theory (perturbatively) is 4-dimensional. Our solution can be embedded in the supergravity context, where we have completely localized supergravity on the corresponding solitonic brane, which is a BPS object preserving $1/2$ of the original supersymmetries. By including a scalar field, we also construct a smooth domain wall solution, which in a certain limit reduces to the δ-function-like solitonic brane solution (this is possible for the latter breaks diffeomorphisms only spontaneously). We then show that in the smooth domain wall background the only normalizable mode is the 4-dimensional graviton zero mode, while all the other (including massive Kaluza-Klein) modes are not even plane-wave normalizable. Finally, we observe that in compactifications of Type IIB on 5-dimensional Einstein manifolds other than a 5-sphere the corresponding dual gauge theories on D3-branes are not conformal in the ultra-violet, and at the quantum level we expect the Einstein-Hilbert term to be generated in their world-volumes. We conjecture that in full string theory on Type IIB side this is due to higher curvature terms, which cannot be ignored in such backgrounds. A stronger version of this conjecture also states that (at least in some cases) in such backgrounds D3-branes are solitonic objects with completely localized (super)gravity in their world-volumes.

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[*]E-mail: iglesias@insti.physics.sunysb.edu

[†]E-mail: zurab@insti.physics.sunysb.edu

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I. INTRODUCTION

In the Brane World scenario the Standard Model gauge and matter fields are assumed to be localized on branes (or an intersection thereof), while gravity lives in a larger dimensional bulk of space-time [1–16]. There is a big difference between the footings on which gauge plus matter fields and gravity come in this picture[1]. Thus, for instance, if gauge and matter fields are localized on D-branes [3], they propagate only in the directions along the D-brane world-volume. Gravity, however, is generically not confined to the branes - even if we have a graviton zero mode localized on the brane as in [14], massive graviton modes are still free to propagate in the bulk.

In this paper we would like to ask the following question. Can we have *complete* localization of gravity? As we will argue in the following, the answer to this question appears to be positive. In particular, in a certain setup, which we will describe in a moment, we will construct a (flat) *solitonic* codimension-one brane world solution, where gravity is completely localized on this solitonic brane. That is, the graviton propagator in the bulk *vanishes*, while it is non-trivial on the brane. In fact, in this solution the gravitational part of the brane world-volume action is given by the usual 4-dimensional Einstein-Hilbert term (assuming that the solitonic brane is a 3-brane). Moreover, even though we have a 5-dimensional theory (in particular, the classical solitonic background is 5-dimensional), the quantum theory (at least perturbatively) is actually 4-dimensional. This is due to the fact that in this solution there are no propagating bulk degrees of freedom, so that there are no loop corrections in the bulk.

The setup within which we construct this solitonic brane world solution is the 5-dimensional Einstein-Hilbert theory with a (negative) cosmological term augmented with a Gauss-Bonnet coupling. The solitonic brane world solution arises in this theory for a special value of the Gauss-Bonnet coupling. The fact that there are no propagating degrees of freedom in the bulk is then due to a perfect cancellation between the corresponding contributions coming from the Einstein-Hilbert and Gauss-Bonnet terms, which occurs precisely for the value of the Gauss-Bonnet coupling for which we have a solitonic brane world solution. Since the bulk theory does not receive loop corrections, this setup (at least perturbatively) is stable at the quantum level as far as 5-dimensional physics is concerned. In particular, the classical choice of parameters such as the special value of the Gauss-Bonnet coupling (or the Gauss-Bonnet combination itself) does *not* require order-by-order fine-tuning.

Since we essentially have a four-dimensional quantum theory, without supersymmetry generically we do expect a quantum instability related to the 4-dimensional cosmological constant. In fact, our solitonic brane world solution does admit curved deformations. However, if we embed this solution in the (minimally) supersymmetric setup, then only the flat solution does not break all the supersymmetries. In fact, the flat solitonic brane world solution is a BPS state which preserves $1/2$ of the original supersymmetries. Moreover, we still have no propagating degrees of freedom in the bulk, while on the brane we have completely localized *supergravity*.

In the aforementioned solitonic brane world solution the brane is δ-function like. Albeit

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[1]This, at least in some sense, might not be an unwelcome feature - see, *e.g.*, [4,7,12].

seemingly strange, this is perfectly consistent as this soliton does not break diffeomorphisms explicitly but *spontaneously*. We should therefore be able to obtain this solitonic brane world solution as a limit of a smooth solitonic brane world. We show that this is indeed the case. We consider the system of Einstein-Hilbert-Gauss-Bonnet gravity coupled to a single scalar field with a non-trivial scalar potential. For a suitable choice of the scalar potential this system possesses kink-like solitonic domain wall solutions, which break diffeomorphisms spontaneously. In a certain limit such a domain wall solution gives precisely the aforementioned solitonic brane world solution. We also point out that in this context the aforementioned special choice of the Gauss-Bonnet coupling required for the solitonic brane world solution to exist is essentially translated into the requirement that the corresponding smooth domain wall solution have 4-dimensional Poincaré invariance.

The aforementioned smooth domain wall solution has the property that only the 4-dimensional graviton zero mode is normalizable in this background (this zero mode is quadratically normalizable). In particular, none of the massive modes are even plane-wave normalizable in this background. This makes it clear why in the limit, where we recover the δ-function-like solitonic brane world solution, gravity is completely localized on the brane. On the other hand, the fact that only the graviton zero mode is normalizable in the smooth domain wall background would not be possible without the higher curvature terms. This indicates that inclusion of higher curvature terms can in some cases qualitatively change gravity in brane world scenarios.

At the end of the paper we speculate on a possible realization of our scenario within string theory. In particular, we observe that in compactifications of Type IIB string theory on X_5, where X_5 is a 5-dimensional Einstein manifold, the dual field theory generically is conformal only in the infra-red but not in the ultra-violet (in the special case where X_5 is a 5-sphere, we have the $\mathcal{N} = 4$ SYM theory which is scale invariant). In such cases we therefore expect that quantum corrections will generically generate [15,16] (among other terms) the Einstein-Hilbert term in the world-volume of the corresponding D3-branes. We conjecture that in the dual Type IIB picture the appearance of this term is due to *higher curvature* terms which should be present in such backgrounds (which have reduced number of unbroken supersymmetries). That is, we suggest that higher curvature terms are important in such backgrounds, and should not be ignored. We also propose a stronger version of this conjecture according to which in full string theory D3-branes with such non-conformal theories in their world-volumes (at least in some cases) are solitonic objects similar to the solitonic brane world we discuss in this paper, and we have completely localized (super)gravity (plus (super)Yang-Mills) on the branes. If this conjecture indeed holds, our scenario might have interesting phenomenological implications such as at least a partial solution to the moduli problem in string compactifications, as well as a possibility of having truly 4-dimensional gravity localized on a brane in non-compact extra space.

II. THE SETUP

In this section we discuss the setup within which we will discuss the aforementioned solitonic brane world solution. The action for this model is given by (for calculational convenience we will keep the number of space-time dimensions D unspecified):

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$$S = M_P^{D-2} \int d^D x \sqrt{-G} \left\{ R + \lambda \left[R^2 - 4R_{MN}^2 + R_{MNST}^2 \right] - \Lambda \right\} , \tag{1}$$

where M_P is the D-dimensional (reduced) Planck scale, and the Gauss-Bonnet coupling λ has dimension (length)^2. Finally, the bulk vacuum energy density Λ is a constant.

The equations of motion following form the action (1) read:

$$R_{MN} - \frac{1}{2}G_{MN}R - \frac{1}{2}\lambda G_{MN} \left(R^2 - 4R^{MN}R_{MN} + R^{MNRS}R_{MNRS} \right) +$$

$$2\lambda \left(RR_{MN} - 2R_{MS}R^S{}_N + R_{MRST}R_N{}^{RST} - 2R^{RS}R_{MRNS} \right) + \frac{1}{2}G_{MN}\Lambda = 0 . \tag{2}$$

Note that this equation does not contain terms with third and fourth derivatives of the metric.

In the following we will be interested in solutions to the above equations of motion with the warped [17] metric of the form

$$ds_D^2 = \exp(2A)\eta_{MN}dx^M dx^N , \tag{3}$$

where η_{MN} is the flat D-dimensional Minkowski metric, and the warp factor A, which is a function of $z \equiv x^D$, is independent of the other $(D-1)$ coordinates x^μ. With this ansätz, we have the following equations of motion for A (prime denotes derivative w.r.t. z):

$$(D-1)(D-2)(A')^2 \left[1 - (D-3)(D-4)\lambda(A')^2 \exp(-2A) \right] + \Lambda \exp(2A) = 0 , \tag{4}$$

$$(D-2) \left[A'' - (A')^2 \right] \left[1 - 2(D-3)(D-4)\lambda(A')^2 \exp(-2A) \right] = 0 . \tag{5}$$

This system of equations has a set of solutions where the D-dimensional space is an AdS space for a continuous range of parameters Λ and λ. The volume of the z direction for this set of solutions is infinite.

There, however, also exists a solution where the volume of the z direction is finite if we "fine-tune" the Gauss-Bonnet coupling λ and the bulk vacuum energy density Λ as follows[2]:

$$\Lambda = -\frac{(D-1)(D-2)}{(D-3)(D-4)}\frac{1}{4\lambda} , \tag{6}$$

where $\lambda > 0$, and $\Lambda < 0$. This solution is given by (we have chosen the integration constant such that $A(0) = 0$):

$$A(z) = -\ln \left[\frac{|z|}{\Delta} + 1 \right] , \tag{7}$$

where Δ is given by

$$\Delta^2 = 2(D-3)(D-4)\lambda . \tag{8}$$

Note that Δ can be positive or negative. In the former case the volume of the z direction is finite: $v = 2\Delta/(D-1)$. On the other hand, in the latter case it is infinite. As we will see in the following, the negative Δ case corresponds to a non-unitary theory.

Note that A' is discontinuous at $z = 0$, and A'' has a δ-function-like behavior at $z = 0$. Note, however, that (5) is still satisfied as in this solution

$$1 - 2(D-3)(D-4)\lambda(A')^2 \exp(-2A) = 0 \; . \tag{9}$$

Thus, this solution describes a codimension one soliton. The tension of this soliton, which is given by

$$f = \frac{4(D-2)}{\Delta} M_P^{D-2} \; , \tag{10}$$

is positive for $\Delta > 0$, and it is negative for $\Delta < 0$. The aforementioned non-unitarity in the latter case is, in fact, attributed to the negativity of the brane tension. Here and in the following we refer to the $z = 0$ hypersurface, call it Σ, as the brane.

III. GRAVITY IN THE SOLITONIC BRANE WORLD

In this section we would like to study gravity in the solitonic brane world solution discussed in the previous section. To do this, let us study small fluctuations around the solution:

$$G_{MN} = \exp(2A) \left[\eta_{MN} + \widetilde{h}_{MN} \right] \; , \tag{11}$$

where for convenience reasons we have chosen to work with $\widetilde{h}_{MN}$ instead of metric fluctuations $h_{MN} = \exp(2A)\widetilde{h}_{MN}$.

To proceed further, we need equations of motion for $\widetilde{h}_{MN}$. Let us assume that we have matter localized on the brane, and let the corresponding conserved energy-momentum tensor be $T_{\mu\nu}$:

$$\partial^\mu T_{\mu\nu} = 0 \; . \tag{12}$$

The graviton field $\widetilde{h}_{\mu\nu}$ couples to $T_{\mu\nu}$ via the following term in the action (note that $\widetilde{h}_{\mu\nu} = h_{\mu\nu}$ at $z = 0$ as we have set $A(0) = 0$):

$$S_{\text{int}} = \frac{1}{2} \int_\Sigma d^{D-1}x \; T_{\mu\nu} \widetilde{h}^{\mu\nu} \; . \tag{13}$$

In the following we will use the following notations for the component fields:

$$H_{\mu\nu} \equiv \widetilde{h}_{\mu\nu} \; , \quad A_\mu \equiv \widetilde{h}_{\mu D} \; , \quad \rho \equiv \widetilde{h}_{DD} \; . \tag{14}$$

The linearized equations of motion for the component fields $H_{\mu\nu}$, A_μ and ρ read:

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$$\left[1 - 2(D-3)(D-4)\lambda(A')^2 \exp(-2A)\right]\left(\partial_\sigma\partial^\sigma H_{\mu\nu} + \partial_\mu\partial_\nu H - \partial_\mu\partial^\sigma H_{\sigma\nu} - \partial_\nu\partial^\sigma H_{\sigma\mu} - \right.$$

$$\eta_{\mu\nu}\left[\partial_\sigma\partial^\sigma H - \partial^\sigma\partial^\rho H_{\sigma\rho}\right] + H''_{\mu\nu} - \eta_{\mu\nu}H'' + (D-2)A'\left[H'_{\mu\nu} - \eta_{\mu\nu}H'\right] -$$

$$\left\{\partial_\mu A'_\nu + \partial_\nu A'_\mu - 2\eta_{\mu\nu}\partial^\sigma A'_\sigma + (D-2)A'\left[\partial_\mu A_\nu + \partial_\nu A_\mu - 2\eta_{\mu\nu}\partial^\sigma A_\sigma\right]\right\} +$$

$$\left.\left\{\partial_\mu\partial_\nu\rho - \eta_{\mu\nu}\partial_\sigma\partial^\sigma\rho + \eta_{\mu\nu}\left[(D-2)A'\rho' + (D-1)(D-2)(A')^2\rho\right]\right\}\right) -$$

$$4(D-4)\lambda\left[A'' - (A')^2\right]\exp(-2A)\left(\partial_\sigma\partial^\sigma H_{\mu\nu} + \partial_\mu\partial_\nu H - \partial_\mu\partial^\sigma H_{\sigma\nu} - \partial_\nu\partial^\sigma H_{\sigma\mu} - \right.$$

$$\left.\eta_{\mu\nu}\left[\partial_\sigma\partial^\sigma H - \partial^\sigma\partial^\rho H_{\sigma\rho}\right] + (D-3)A'\left\{H'_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu - \eta_{\mu\nu}\left[H' - 2\partial^\sigma A_\sigma\right]\right\}\right) +$$

$$2(D-2)\left[A'' - (A')^2\right]\left[1 - 4(D-3)(D-4)\lambda(A')^2\exp(-2A)\right]\eta_{\mu\nu}\rho =$$

$$-M_P^{2-D}T_{\mu\nu}\delta(z) , \tag{15}$$

$$\left[1 - 2(D-3)(D-4)\lambda(A')^2\exp(-2A)\right]\left(\left[\partial^\mu H_{\mu\nu} - \partial_\nu H\right]' - \partial^\mu F_{\mu\nu} + \right.$$

$$\left.(D-2)A'\partial_\nu\rho\right) = 0 , \tag{16}$$

$$\left[1 - 2(D-3)(D-4)\lambda(A')^2\exp(-2A)\right]\left(-\left[\partial^\mu\partial^\nu H_{\mu\nu} - \partial^\mu\partial_\mu H\right] + \right.$$

$$\left.(D-2)A'\left[H' - 2\partial^\sigma A_\sigma\right] - (D-1)(D-2)(A')^2\rho\right) = 0 , \tag{17}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the $U(1)$ field strength for the graviphoton, and $H \equiv H_\mu{}^\mu$.

The above equations of motion are invariant under certain gauge transformations corresponding to unbroken diffeomorphisms. In terms of $\widetilde{h}_{MN}$ the full D-dimensional diffeomorphisms

$$\delta h_{MN} = \nabla_M\xi_N + \nabla_N\xi_M \tag{18}$$

are given by the following gauge transformations (here we use $\xi_M \equiv \exp(2A)\widetilde{\xi}_M$):

$$\delta\widetilde{h}_{MN} = \partial_M\widetilde{\xi}_N + \partial_N\widetilde{\xi}_M + 2A'\eta_{MN}\omega , \tag{19}$$

where $\omega \equiv \widetilde{\xi}_D$. In terms of the component fields $H_{\mu\nu}$, A_μ and ρ, the full D-dimensional diffeomorphisms read:

$$\delta H_{\mu\nu} = \partial_\mu\widetilde{\xi}_\nu + \partial_\nu\widetilde{\xi}_\mu + 2\eta_{\mu\nu}A'\omega , \tag{20}$$

$$\delta A_\mu = \widetilde{\xi}'_\mu + \partial_\mu\omega , \tag{21}$$

$$\delta\rho = 2\omega' + 2A'\omega . \tag{22}$$

It is not difficult to check that the equations of motion (15), (16) and (17) are invariant under these full D-dimensional diffeomorphisms. That is, there are no restrictions on ω or $\widetilde{\xi}_\mu$ or derivatives thereof including at $z = 0$. In particular, this is the case for the solitonic brane world solution despite its δ-function-like structure. The reason for this is that this solution being a soliton does not break the full D-dimensional diffeomorphisms explicitly but only *spontaneously*.

Since we have the full D-dimensional diffeomorphisms, we can always gauge A_μ and ρ away. In fact, in the following we will see that for the solitonic brane world background this can indeed be done without introducing any inconsistencies. However, before we adapt this gauge fixing and solve the above equations of motion, we would like to make the following

important observation. Note that for the solitonic brane world solution (7) with Δ given by (8) we have (9). On the other hand, this vanishing factor is precisely the one that multiplies the terms in (15), (16) and (17) corresponding to the propagation of the fields $H_{\mu\nu}$, A_μ and ρ in the bulk. That is, in the solitonic brane world solution these fields do not propagate in the z direction at all. This is due to a cancellation between contributions of the Einstein-Hilbert and Gauss-Bonnet terms into the bulk propagator in this background[3]. On the other hand, (some of) these fields do propagate on the brane. Indeed, in the above background we have

$$A'' - (A')^2 = -\frac{2}{\Delta}\delta(z) \ . \tag{23}$$

Then (15) gives the following equation of motion (note that (16) and (17) are trivially satisfied in this background):

$$
\begin{aligned}
&\Big(\partial_\sigma\partial^\sigma H_{\mu\nu} + \partial_\mu\partial_\nu H - \partial_\mu\partial^\sigma H_{\sigma\nu} - \partial_\nu\partial^\sigma H_{\sigma\mu} - \eta_{\mu\nu}\left[\partial_\sigma\partial^\sigma H - \partial^\sigma\partial^\rho H_{\sigma\rho}\right] + \\
&(D-3)A'\left\{H'_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu - \eta_{\mu\nu}\left[H' - 2\partial^\sigma A_\sigma\right]\right\} + (D-2)(D-3)\Delta^{-2}\eta_{\mu\nu}\rho\Big)\delta(z) = \\
&-\widehat{M}_P^{3-D}T_{\mu\nu}\delta(z) \ ,
\end{aligned}
\tag{24}
$$

where

$$\widehat{M}_P^{D-3} \equiv \frac{4\Delta}{D-3}M_P^{D-2} \ , \tag{25}$$

and in the following we will identify $\widehat{M}_P$ with the $(D-1)$-dimensional Planck scale.

A. Completely Localized Gravity

Next, we would like to see what is the solution to the equation of motion (24). First, note that, as we have already mentioned, we can always gauge A_μ and ρ away. That is, these fields are *not* propagating degrees of freedom. Note that after this gauge fixing the residual gauge symmetry is given by the $(D-1)$-dimensional diffeomorphisms for which $\omega \equiv 0$, and $\tilde{\xi}_\mu$ are independent of z. Second, note that the term in the curly brackets in (24) is multiplied by $A'\delta(z)$. This quantity, however, is vanishing as A' has a sign(z)-like discontinuity at $z = 0$. We therefore obtain the following equation of motion for the $(D-1)$-dimensional graviton components $H_{\mu\nu}$:

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[3]Note that in (15), (16) and (17) the terms that survive in the limit where the warp factor A is a constant correspond to the terms that arise upon linearization of the D-dimensional Einstein-Hilbert term around the *flat* background. On the other hand, there are no such terms corresponding to linearization of the D-dimensional Gauss-Bonnet term around the flat background. This is due to the fact that even in $D > 4$ the terms quadratic in metric fluctuations coming from expanding the Gauss-Bonnet term around the flat background give rise to a total derivative in the action [19,20] (in $D = 4$ the Gauss-Bonnet term is a total derivative altogether as it corresponds to the 4-dimensional Euler invariant).

$$\left(\partial_\sigma \partial^\sigma H_{\mu\nu} + \partial_\mu \partial_\nu H - \partial_\mu \partial^\sigma H_{\sigma\nu} - \partial_\nu \partial^\sigma H_{\sigma\mu} - \eta_{\mu\nu} \left[\partial_\sigma \partial^\sigma H - \partial^\sigma \partial^\rho H_{\sigma\rho} \right] + \widehat{M}_P^{3-D} T_{\mu\nu} \right) \delta(z) = 0 \ . \tag{26}$$

Note that this equation is purely $(D-1)$-dimensional. Thus, gravity is *completely* localized on the brane, that is at the $z = 0$ hypersurface Σ. In particular, the graviton field $H_{\mu\nu}$ is non-vanishing only on the brane, while it vanishes in the bulk:

$$H_{\mu\nu}(z \neq 0) = 0 \ . \tag{27}$$

Note that (26) does not by itself imply (27). In particular, *a priori* $H_{\mu\nu}$ at $z \neq 0$ can be arbitrary. However, as we explained above, we have no propagating degrees of freedom in the bulk, that is, the graviton propagator in the bulk vanishes, while it is non-vanishing only on the brane. This implies that perturbations due to matter localized on the brane should *not* propagate into the bulk but only on the brane, hence (27).

On the brane (26) can be solved in a standard way. Thus, in the harmonic gauge (we can use this or any other suitable gauge fixing on the brane as we have unbroken $(D-1)$-dimensional diffeomorphisms)

$$\partial^\mu H_{\mu\nu} = \frac{1}{2} \partial_\nu H \tag{28}$$

we have

$$H_{\mu\nu}(p, z = 0) = \widehat{M}_P^{3-D} \frac{1}{p^2} \left[T_{\mu\nu}(p) - \frac{1}{D-3} \eta_{\mu\nu} T(p) \right] \ , \tag{29}$$

where we have performed the Fourier transform w.r.t. the $(D-1)$-dimensional coordinates x^μ (the corresponding momenta are p^μ, and $p^2 \equiv p^\mu p_\mu$), and $T(p) \equiv T_\mu{}^\mu(p)$. From (26) as well as (29) it is clear that $\widehat{M}_P$ is the $(D-1)$-dimensional Planck scale for $(D-1)$-dimensional gravity localized on the brane (note that the momentum and tensor structures in (29) are $(D-1)$-dimensional). Actually, $\widehat{M}_P$ is identified with the $(D-1)$-dimensional Planck scale for the positive Δ solution. As to the negative Δ solution, we have "antigravity" localized on the brane, and the corresponding theory is non-unitary due to negative norm states propagating on the brane.

Note that above our analysis was confined to the linearized theory. The above conclusions, however, are valid in the full non-linear theory. Indeed, we have no propagating degrees of freedom in the bulk, while on the brane we have only the zero mode for the $(D-1)$-dimensional graviton components $H_{\mu\nu}$. This then implies that in the solitonic brane world background (the gravitational part of) the brane world-volume theory is described by the $(D-1)$-dimensional Einstein-Hilbert action:

$$S_{\text{brane}} = \widehat{M}_P^{D-2} \int d^{D-1}x \sqrt{-\widehat{G}} \widehat{R} \ , \tag{30}$$

where $\widehat{G}_{\mu\nu}$ is the $(D-1)$-dimensional metric on the brane; all the hatted quantities are $(D-1)$-dimensional, and are constructed from $\widehat{G}_{\mu\nu}$. Note that there is no $(D-1)$-dimensional Gauss-Bonnet term in this action, which can be seen by examining (2).

IV. QUANTUM STABILITY

The solitonic brane world solution we discussed in the previous sections has the following remarkable property - the bulk theory does not receive any loop corrections. Indeed, there are no propagating degrees of freedom in the bulk, hence the absence of loop corrections[4]. This implies that the bulk action is not renormalized at all, and, in particular, the relation (6) between the bulk vacuum energy density Λ and the Gauss-Bonnet coupling λ is stable against quantum corrections. This is why we used the word "fine-tuning" in quotation marks in section II - once we choose the parameters of the classical theory to satisfy (6), we need no fine-tuning at the quantum level.

The above observation has important implications for gravity in the solitonic brane world solution. First, there is no danger of delocalization of gravity, which is generically expected to occur at the quantum level due to higher curvature bulk terms [21–23] in warped backgrounds such as [14]. Second, due to the spontaneous nature of diffeomorphism breaking in the solitonic brane world, the graviscalar and graviphoton components are pure gauge degrees of freedom. This implies that at the quantum level there is no danger of generating brane world-volume terms involving, say, the graviscalar, which are generically expected to lead to inconsistencies in the coupling between bulk gravity and brane matter [22] in warped backgrounds such as [14]. These properties of the solitonic brane world can be understood in a simple way by noting that the quantum theory is actually $(D-1)$-dimensional (albeit the classical background is D-dimensional), so the only quantum instability we can expect is that related to the $(D-1)$-dimensional physics.

A. Curved Deformations

Such an instability generically indeed exists as we are dealing with a theory containing gravity - without supersymmetry we expect that generically $(D-1)$-dimensional cosmological constant will be generated at the quantum level. Here we would like to verify that the solitonic brane world indeed admits curved deformations.

Thus, instead of the flat ansätz (3), let us look for solutions with the following warped metric

$$ds_D^2 = \exp(2A) \left[\widehat{g}_{\mu\nu} dx^\mu dx^\nu + (dz)^2 \right] \ , \tag{31}$$

where the $(D-1)$-dimensional background metric $\widehat{g}_{\mu\nu}$ is independent of z, but need not be flat. The equations of motion (2) then give the following equations of motion for A:

$$(D-1)(D-2)(A')^2 \left[1 - (D-3)(D-4)\lambda(A')^2 \exp(-2A) \right] + \Lambda \exp(2A) -$$
$$\frac{D-1}{D-3}\widehat{\Lambda} \left[1 - 2(D-3)(D-4)\lambda(A')^2 \exp(-2A) \right] - \lambda\widehat{\chi} \exp(-2A) = 0 \ , \tag{32}$$

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[4]From now on, when referring to quantum stability or absence of quantum corrections, we mean perturbatively. *A priori* there might be non-perturbative corrections in the bulk which might modify some of the following conclusions, but this issue is outside of the scope of this paper.

$$(D-2)\left[A'' - (A')^2\right]\left[1 - 2(D-3)(D-4)\lambda(A')^2\exp(-2A) + 2\frac{D-4}{D-2}\lambda\widehat{\Lambda}\exp(-2A)\right] +$$

$$\frac{1}{D-3}\widehat{\Lambda}\left[1 - 2(D-3)(D-4)\lambda(A')^2\exp(-2A)\right] + \frac{2\lambda}{D-1}\widehat{\chi}\exp(-2A) = 0 \ . \tag{33}$$

Here $\widehat{\Lambda}$ is the cosmological constant of the $(D-1)$-dimensional manifold, which is therefore an Einstein manifold, described by the metric $\widehat{g}_{\mu\nu}$. Our normalization of $\widehat{\Lambda}$ is such that the $(D-1)$-dimensional metric $\widehat{g}_{\mu\nu}$ satisfies Einstein's equations

$$\widehat{R}_{\mu\nu} - \frac{1}{2}\widehat{g}_{\mu\nu}\widehat{R} = -\frac{1}{2}\widehat{g}_{\mu\nu}\widehat{\Lambda} \ . \tag{34}$$

Moreover, the quantity

$$\widehat{\chi} \equiv \widehat{R}^2 - 4\widehat{R}_{\mu\nu}^2 + \widehat{R}_{\mu\nu\sigma\tau}^2 \tag{35}$$

is also a constant (for $\lambda \neq 0$). Finally, the aforementioned $(D-1)$-dimensional Einstein manifold must be such that

$$\widehat{R}_{\mu\rho\sigma\tau}\widehat{R}_\nu{}^{\rho\sigma\tau} = \frac{1}{D-1}\widehat{R}_{\lambda\rho\sigma\tau}^2\widehat{g}_{\mu\nu} \ . \tag{36}$$

This condition is automatically satisfied for maximally symmetric Einstein manifolds.

Albeit the Einstein manifold described by the metric $\widehat{g}_{\mu\nu}$ *a priori* need not be maximally symmetric, for our purposes here it will suffice to confine our attention to maximally symmetric cases. Then we have

$$\widehat{R}_{\mu\nu\rho\sigma} = \frac{\widehat{\Lambda}}{(D-2)(D-3)}\left(\widehat{g}_{\mu\rho}\widehat{g}_{\nu\sigma} - \widehat{g}_{\mu\sigma}\widehat{g}_{\nu\rho}\right) \ , \tag{37}$$

and

$$\widehat{\chi} = \frac{(D-1)(D-4)}{(D-2)(D-3)}\widehat{\Lambda}^2 \ . \tag{38}$$

It is then not difficult to see that, for the special value of the bulk vacuum energy density given by (6), the second order equation (33) is automatically satisfied as long as the first order equation (32) is satisfied. In fact, this latter equation simplifies as follows:

$$(A')^2 = \frac{\widehat{\Lambda}}{(D-2)(D-3)} + \frac{1}{\Delta^2}\exp(2A) \ , \tag{39}$$

where Δ is given by (8). This equation has consistent smooth solutions (that is, solutions with continuous derivatives of A) for any $\widehat{\Lambda}$. These solutions correspond to foliations of the D-dimensional AdS space with the vacuum energy density $\Lambda_1 = 2\Lambda$ with maximally symmetric $(D-1)$-dimensional Einstein manifolds. On the other hand, as in the flat case, we also have solitonic brane world solutions. Thus, let $A_1(z)$ be a smooth solution of the aforementioned type. Then the solution

$$A(z) = A_1(|z|) \tag{40}$$

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describes a solitonic brane world solution. If at $z \to +\infty$ we have $A_1(z) \to -\infty$, then (40) gives a curved deformation of a flat solution with positive brane tension. If at $z \to +\infty$ we have $A_1(z) \to +\infty$, then (40) gives a curved deformation of a flat solution with negative brane tension. The former type of solutions give a consistent curved solitonic brane world.

Here the following remark is in order. The solitonic brane world solutions obtained via the above procedure have discontinuous A' for generic values of $\widehat{\Lambda}$. There is, however, a special value of $\widehat{\Lambda}$, given by

$$\widehat{\Lambda} = -(D-2)(D-3)\Delta^{-2} , \tag{41}$$

for which $A_1(z)$ is symmetric w.r.t. the reflection $z \to -z$, so that $A(z)$ has a continuous derivative everywhere. That is, the solitonic brane world solution given by (40) in this case coincides with the corresponding smooth solution. From now on, when referring to the solitonic brane world solution, we will therefore *not* include the case where $\widehat{\Lambda}$ satisfies (41).

B. Supersymmetry and BPS Solitonic Brane World

Thus, as expected, *a priori* we can have non-vanishing cosmological constant on the above solitonic brane. However, if we embed this solution in the supersymmetric setup, then only the flat solution does not break all the supersymmetries[5]. Here we would like to discuss this supersymmetric generalization in more detail.

Note that the action (1) can be supersymmetrized (for the standard values of D). Upon supersymmetrization, we have fields other than the metric. Let us consider backgrounds where all bosonic fields other than the metric have vanishing expectation values. Then the solitonic brane world solution is the same as before. Since graviton does not propagate in the bulk in this background, it then follows by supersymmetry that no other fields will have propagating degrees of freedom in the bulk either (here we are assuming that we have minimal supersymmetry). That is, the cancellation between the bulk Einstein-Hilbert and Gauss-Bonnet terms that we saw above generalizes to all the other (superpartner) terms in the bulk as well. On the other hand, on the brane we do have propagating degrees of freedom. In fact, we expect to have completely localized *supergravity* on the brane if there are some unbroken supersymmetries. Here we would like to show that $\widehat{\Lambda} \neq 0$ solutions break all supersymmetries (regardless of the sign of $\widehat{\Lambda}$), while the flat solitonic brane world is a BPS solution which preserves $1/2$ of the original supersymmetries.

To see this, let us study Killing spinors in the solitonic brane world background:

$$\mathcal{D}_M \epsilon = 0 . \tag{42}$$

Here $\mathcal{D}_M$ is a generalized covariant derivative:

$$\mathcal{D}_M = D_M - \frac{1}{2}W\Gamma_M , \tag{43}$$

---

[5]As we pointed out in the previous subsection, if the cosmological constant on the brane satisfies (41), then the corresponding solution is actually smooth. In this case all the supersymmetries are therefore intact, but we do not interpret this solution as the solitonic brane world solution.

where D_M is the usual covariant derivative containing the spin connection, and W is interpreted as the superpotential. The D-dimensional gamma matrices Γ_M are defined via

$$\{\Gamma_M , \Gamma_N\} = 2g_{MN} , \tag{44}$$

where $g_{MN} = \exp(2A)\widetilde{g}_{MN}$ is the background metric: $\widetilde{g}_{\mu\nu} = \widehat{g}_{\mu\nu}$, $\widetilde{g}_{\mu D} = 0$, and $\widetilde{g}_{DD} = 1$.

It is not difficult to show that in such warped backgrounds we have

$$0 = \mathcal{D}_D\epsilon = \epsilon' - \frac{1}{2}W \exp(A)\widetilde{\Gamma}_D\epsilon , \tag{45}$$

$$0 = \mathcal{D}_\mu\epsilon = \widehat{D}_\mu\epsilon + \frac{1}{2}\widehat{\Gamma}_\mu \left[A'\widetilde{\Gamma}_D - W \exp(A)\right] \epsilon , \tag{46}$$

where $\widehat{D}_\mu$ is the $(D-1)$-dimensional covariant derivative corresponding to the metric $\widehat{g}_{\mu\nu}$, while $\widehat{\Gamma}_\mu = \widetilde{\Gamma}_\mu \equiv \exp(-A)\Gamma_\mu$ are the $(D-1)$-dimensional gamma matrices corresponding to the metric $\widehat{g}_{\mu\nu}$, and $\widetilde{\Gamma}_D \equiv \exp(-A)\Gamma_D$ is a constant matrix.

Before solving the Killing spinor equations, let us note that to define an unbroken supercharge for a given Killing spinor we must make sure that the global integrability conditions

$$[\mathcal{D}_M , \mathcal{D}_N]\epsilon = 0 \tag{47}$$

are also satisfied. In the component form these conditions read:

$$0 = [\mathcal{D}_\mu , \mathcal{D}_\nu]\epsilon = \frac{1}{4}\left(\frac{1}{2}\widehat{R}_{\mu\nu\sigma\tau}\left[\widehat{\Gamma}^\sigma , \widehat{\Gamma}^\tau\right] + \left[W^2\exp(2A) - (A')^2\right]\left[\widehat{\Gamma}_\mu , \widehat{\Gamma}_\nu\right]\right)\epsilon , \tag{48}$$

$$0 = [\mathcal{D}_\mu , \mathcal{D}_D]\epsilon = \frac{1}{2}\widehat{\Gamma}_\mu \left(W'\exp(A) + \left[W^2\exp(2A) - A''\right]\widetilde{\Gamma}_D\right)\epsilon . \tag{49}$$

Since in the solitonic brane world solution A' is discontinuous, to satisfy the last condition W must be discontinuous as well. Then only $1/2$ of supersymmetries can be preserved, and the corresponding Killing spinor has a definite helicity w.r.t. $\widetilde{\Gamma}_D$:

$$\widetilde{\Gamma}_D\epsilon = \eta\epsilon , \tag{50}$$

where η is either $+1$ or -1. It is then not difficult to see that, to have a non-trivial solution to (46) compatible with the condition (48), the cosmological constant on the brane must be vanishing, and we have the following BPS equation:

$$A' = \eta W \exp(A) . \tag{51}$$

This equation together with the solution (7) then implies that

$$W = -\eta\Delta^{-1}\mathrm{sign}(z) . \tag{52}$$

The Killing spinor is then given by

$$\epsilon = \exp\left[\frac{1}{2}A\right]\epsilon_0 , \tag{53}$$

where ϵ_0 is a *constant* spinor with helicity η:

$$\widetilde{\Gamma}_D\epsilon_0 = \eta\epsilon_0 . \tag{54}$$

Thus, as we see, the flat solution is a BPS solution preserving $1/2$ of supersymmetries.

The BPS solitonic brane world solution is now stable against quantum corrections on the brane - the cosmological constant on the brane is vanishing as long as supersymmetry is unbroken.

V. SOLITONIC BRANE WORLD AS A LIMIT OF A SMOOTH DOMAIN WALL

In the solitonic brane world solution we discussed in the previous sections the brane is δ-function like. It might appear strange that such a brane is a soliton. In this section, however, we show that our solitonic brane can be obtained as a certain limit of a smooth domain wall solution. Here we would like to emphasize that this is possible as our solitonic brane world solution does not break diffeomorphisms explicitly but spontaneously[6].

A. Setup

In this subsection we discuss the setup within which we will construct a smooth solution whose limit gives the aforementioned solitonic brane world. In parts of the remainder of this section we closely follow discussion in [23] (also see [25]). Thus, consider a single real scalar field ϕ coupled to gravity with the following action[7]:

$$
S = M_P^{D-2} \int d^D x \sqrt{-G} \Big[R + \lambda \left(R^2 - 4R^{MN} R_{MN} + R^{MNRS} R_{MNRS} \right)
$$
$$
- \frac{4}{D-2} (\nabla \phi)^2 - V(\phi) \Big] , \tag{55}
$$

where $V(\phi)$ is the scalar potential for ϕ. The equations of motion read:

$$
\frac{8}{D-2} \nabla^2 \phi = V_\phi , \tag{56}
$$

$$
R_{MN} - \frac{1}{2} G_{MN} R - \frac{1}{2} \lambda G_{MN} \left(R^2 - 4R^{MN} R_{MN} + R^{MNRS} R_{MNRS} \right) +
$$
$$
2\lambda \left(RR_{MN} - 2R_{MS} R^S{}_N + R_{MRST} R_N{}^{RST} - 2R^{RS} R_{MRNS} \right)
$$
$$
= \frac{4}{D-2} \left[\nabla_M \phi \nabla_N \phi - \frac{1}{2} G_{MN} (\nabla \phi)^2 \right] - \frac{1}{2} G_{MN} V . \tag{57}
$$

The subscript ϕ in V_ϕ denotes derivative w.r.t. ϕ.

---

[6]This is an important point. Thus, note that if we introduce a δ-function-like brane source by hand as, say, in [14], some diffeomorphisms are *explicitly* broken. Such non-solitonic brane then cannot be thought of as a limit of a smooth domain wall solution as the latter breaks diffeomorphisms spontaneously [21]. Indeed, diffeomorphisms are a local symmetry in this context, so there is a discontinuity between spontaneously *vs.* explicitly broken diffeomorphisms just as there is a discontinuity between, say, spontaneously *vs.* explicitly broken $U(1)$ gauge symmetry. This as well as other related issues will be discussed in more detail in [24].

[7]Here we focus on the case with one scalar field for the sake of simplicity. In particular, in this case we can absorb a (non-singular) metric $Z(\phi)$ in the $(\nabla \phi)^2$ term by a non-linear field redefinition. This cannot generically be done in the case of multiple scalar fields ϕ^i, where one must therefore also consider the metric $Z_{ij}(\phi)$.

In the following we will be interested in solutions to the above equations of motion where the metric has the form

$$ds^2 = \exp(2A)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \ , \tag{58}$$

and the warp factor A and the scalar field ϕ are non-trivial functions of y but are independent of the x^μ coordinates. Here for convenience reasons we choose to work in the coordinate system (x^μ, y) instead of (x^μ, z) as in (3). The two coordinate systems are related via

$$dy = \exp(A)dz \ . \tag{59}$$

With this ansätz we have the following equations of motion for ϕ and A:

$$\frac{8}{D-2}\left[\phi_{yy} + (D-1)A_y\phi_y\right] - V_\phi = 0 \ , \tag{60}$$

$$(D-1)(D-2)(A_y)^2\left[1 - (D-3)(D-4)\lambda(A_y)^2\right] - \frac{4}{D-2}(\phi_y)^2 + V = 0 \ , \tag{61}$$

$$(D-2)A_{yy}\left[1 - 2(D-3)(D-4)\lambda(A_y)^2\right] + \frac{4}{D-2}(\phi_y)^2 = 0 \ , \tag{62}$$

where a subscript y denotes derivative w.r.t. y. We can rewrite these equations in terms of the following first order equations

$$\phi_y = \alpha w_\phi \left(1 - \lambda\kappa w^2\right) \ , \tag{63}$$

$$A_y = \beta w \ , \tag{64}$$

where

$$\alpha \equiv \sigma\frac{\sqrt{D-2}}{2} \ , \tag{65}$$

$$\beta \equiv -\sigma\frac{2}{(D-2)^{3/2}} \ , \tag{66}$$

$$\kappa \equiv 2(D-3)(D-4)\beta^2 \ , \tag{67}$$

and $\sigma = \pm 1$. Moreover, the scalar potential V is related to the function $w = w(\phi)$ via

$$V = \left[w_\phi^2 + \Omega\right]\left(1 - \lambda\kappa w^2\right)^2 - \Omega \ , \tag{68}$$

where

$$\Omega \equiv \frac{(D-1)(D-2)}{4\lambda(D-3)(D-4)} \ . \tag{69}$$

Note that for $\lambda > 0$ the potential (68) is bounded below [25]. Also note that in the $\lambda \to 0$ limit from (68) we recover the familiar expression $V = w_\phi^2 - \gamma^2 w^2$, where $\gamma^2 \equiv 4(D-1)/(D-2)^2$.

B. A Domain Wall Solution and the Limit

Next, we would like to give a simple example of a domain wall solution to the above equations of motion. Thus, let us assume that $\lambda > 0$, and let

$$w = \zeta\phi \ . \tag{70}$$

The domain wall solution is then given by:

$$\phi(y) = \frac{1}{\zeta\sqrt{\lambda\kappa}} \tanh\left[\alpha\zeta^2\sqrt{\lambda\kappa}(y - y_0)\right] \ , \tag{71}$$

$$A(y) = \frac{\beta}{\alpha\zeta^2\lambda\kappa} \ln\left(\cosh\left[\alpha\zeta^2\sqrt{\lambda\kappa}(y - y_0)\right]\right) + A_0 \ , \tag{72}$$

where y_0 and A_0 are integration constants, which we will set to zero in the following. Then the point $y = 0$ corresponds to the "center" of the domain wall, and at this point the warp factor vanishes: $A(0) = 0$.

Note that in this solution the volume of the y direction, which is given by

$$v = \int dy \exp[(D - 1)A] \ , \tag{73}$$

is finite. This implies that gravity is localized on the domain wall.

Next, consider the limit where the parameter $\zeta \to \infty$. In this limit the scalar $\phi(y)$ vanishes everywhere, while the warp factor $A(y)$ is given by:

$$A(y) = -|y|/\Delta \ , \tag{74}$$

where Δ is given by (8). If we rewrite this solution in terms of the z coordinate, we will obtain (7). Thus, as we see, the solitonic brane world solution discussed in the previous sections is indeed recovered in the $\zeta \to \infty$ limit of the above smooth domain wall solution. Moreover, the smooth solution interpolates between two AdS minima $\phi_\pm$ of the scalar potential $V(\phi)$, and in these minima we have

$$V(\phi_\pm) = -\Omega \ . \tag{75}$$

Note that this is precisely the "fine-tuned" bulk vacuum energy density (6) for which the solitonic brane world solution exists. In the context of smooth domain wall solutions, however, this does not appear to be a fine-tuning in the following sense. Smooth domain walls interpolate between the points $\phi_\pm$ for which the r.h.s. of (63) is vanishing, that is

$$w_\phi\left[1 - \lambda\kappa w^2\right]\Big|_{\phi_\pm} = 0 \ . \tag{76}$$

Note that *a priori* the function $w(\phi)$ need not have an extremum. Let us assume that this is indeed the case. Then for $\lambda > 0$ *flat* domain wall solutions exist if and only if the equation

$$1 - \lambda\kappa w^2 = 0 \tag{77}$$

has at least two distinct roots (the domain wall then interpolates between a pair of adjacent points $\phi_\pm$ for which (77) is satisfied). Note that our ability to rewrite the equations of motion

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in terms of the first order equations (63) and (64) is due to the fact that we are considering solutions where we have $(D-1)$-dimensional Poincaré invariance on the domain wall. We therefore conclude that in the context of considering the solitonic brane world solution (7) as a limit of a smooth domain wall solution the *classical*[8] "fine-tuning" condition (6) is not any less generic than the fine-tuning of the brane cosmological constant. The latter fine-tuning is absent in the supersymmetric context, so in the above sense the question whether (6) is a classical fine-tuning is reduced to the question whether there is any reason why the function $w(\phi)$ should have (multiple) extrema.

Before we end this subsection, let us make the following remarks. In the supersymmetric context the above smooth domain wall can be shown to be a BPS state which preserves $1/2$ of the original supersymmetries. The function w in this case plays the role of the superpotential, while the first order equations (63) and (64) are the BPS equations. Also, in the $\zeta \to \infty$ limit we have

$$\eta W = \beta w = -\frac{1}{\Delta}\mathrm{sign}(z) \ , \tag{78}$$

where W was introduced in subsection B of the previous section.

Here we should point out that *a priori* there is no guarantee that the aforementioned smooth domain wall solutions should be embeddable in the supersymmetric (that is, supergravity) context for, say, $D > 4$. In particular, without the Gauss-Bonnet term such an embedding appears to be non-trivial [26,27]. Note, however, that the presence of higher curvature terms might have important implications for this issue. Thus, for instance, in the above context the superpotential $w(\phi)$ need not have extrema for the domain wall to exist, while without the Gauss-Bonnet term it must. At any rate, whether or not smooth domain walls of the above type can be embedded in supergravity is outside of the scope of this paper. Note, however, that the solitonic brane world solution discussed in the previous sections is clearly embeddable in the supergravity framework.

C. Normalizable Modes

Let us now study gravity in the above smooth domain wall background. This will help us better understand how come gravity is completely localized in the solitonic brane world solution discussed above. Thus, let us consider small fluctuations around the domain wall solution (71) and (72). Here we are going to be interested in understanding the normalizable modes, so we will not need to include a source term. The linearized equations of motion then read:

$$\left[1 - 2(D-3)(D-4)\lambda(A')^2 \exp(-2A)\right]\left(\partial_\sigma\partial^\sigma H_{\mu\nu} + \partial_\mu\partial_\nu H - \partial_\mu\partial^\sigma H_{\sigma\nu} - \partial_\nu\partial^\sigma H_{\sigma\mu} - \right.$$
$$\eta_{\mu\nu}\left[\partial_\sigma\partial^\sigma H - \partial^\sigma\partial^\rho H_{\sigma\rho}\right] + H''_{\mu\nu} - \eta_{\mu\nu}H'' + (D-2)A'\left[H'_{\mu\nu} - \eta_{\mu\nu}H'\right] -$$
$$\left\{\partial_\mu A'_\nu + \partial_\nu A'_\mu - 2\eta_{\mu\nu}\partial^\sigma A'_\sigma + (D-2)A'\left[\partial_\mu A_\nu + \partial_\nu A_\mu - 2\eta_{\mu\nu}\partial^\sigma A_\sigma\right]\right\} +$$

---

[8]Recall that this condition is stable against loop corrections in the context of solitonic brane world with completely localized gravity.

$$\left\{\partial_\mu\partial_\nu\rho - \eta_{\mu\nu}\partial_\sigma\partial^\sigma\rho + \eta_{\mu\nu}\left[(D-2)A'\rho' + (D-1)(D-2)(A')^2\rho\right]\right\}\right) -$$

$$4(D-4)\lambda\left[A'' - (A')^2\right]\exp(-2A)\left(\partial_\sigma\partial^\sigma H_{\mu\nu} + \partial_\mu\partial_\nu H - \partial_\mu\partial^\sigma H_{\sigma\nu} - \partial_\nu\partial^\sigma H_{\sigma\mu} -\right.$$

$$\eta_{\mu\nu}\left[\partial_\sigma\partial^\sigma H - \partial^\sigma\partial^\rho H_{\sigma\rho}\right] + (D-3)A'\left\{H'_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu - \eta_{\mu\nu}\left[H' - 2\partial^\sigma A_\sigma\right]\right\}\right) +$$

$$\eta_{\mu\nu}\rho\left(2(D-2)\left[A'' - (A')^2\right]\left[1 - 4(D-3)(D-4)\lambda(A')^2\exp(-2A)\right] + \frac{4}{D-2}(\phi')^2\right) =$$

$$\frac{8}{D-2}\eta_{\mu\nu}\phi'\varphi' + \eta_{\mu\nu}\varphi V_\phi\exp(2A) , \tag{79}$$

$$\left[1 - 2(D-3)(D-4)\lambda(A')^2\exp(-2A)\right]\left(\left[\partial^\mu H_{\mu\nu} - \partial_\nu H\right]' - \partial^\mu F_{\mu\nu} +\right.$$

$$(D-2)A'\partial_\nu\rho\right) = \frac{8}{D-2}\phi'\partial_\nu\varphi , \tag{80}$$

$$\left[1 - 2(D-3)(D-4)\lambda(A')^2\exp(-2A)\right]\left(-\left[\partial^\mu\partial^\nu H_{\mu\nu} - \partial^\mu\partial_\mu H\right] +\right.$$

$$(D-2)A'\left[H' - 2\partial^\sigma A_\sigma\right] - (D-1)(D-2)(A')^2\rho\right) + \frac{4}{D-2}(\phi')^2\rho =$$

$$\frac{8}{D-2}\phi'\varphi' - \varphi V_\phi\exp(2A) , \tag{81}$$

$$\partial_\mu\partial^\mu\varphi + \varphi'' + (D-2)A'\varphi' - \frac{D-2}{8}\varphi V_{\phi\phi}\exp(2A) -$$

$$\frac{1}{2}\phi'\left[2\partial^\mu A_\mu + \rho' - H'\right] - \frac{D-2}{8}\rho V_\phi\exp(2A) = 0 , \tag{82}$$

where φ is the fluctuation for the scalar field ϕ, and we are working in the (x^μ, z) coordinate system.

Since the domain wall breaks diffeomorphisms only spontaneously, we can use the full D-dimensional diffeomorphisms (20), (21) and (22) to simplify these equations of motion. In fact, such a simplification indeed takes place if we perform the gauge transformation with $\widetilde{\xi}_\mu \equiv 0$ and (note that ϕ' is non-vanishing everywhere) [21,28]

$$\omega = -\varphi/\phi' . \tag{83}$$

It is not difficult to check that this gauge transformation simply removes φ from the above equations of motion. Indeed, under the diffeomorphisms we have

$$\delta\varphi = \phi'\omega . \tag{84}$$

That is, φ is *not* a propagating degree of freedom in this gauge [21,28]. Note that this uses up some diffeomorphisms, but the residual diffeomorphisms are sufficient to also gauge A_μ away. Indeed, after we remove φ from the equations of motion, we can use the diffeomorphisms with $\omega \equiv 0$ but non-trivial $\widetilde{\xi}_\mu$ to set A_μ to zero without otherwise changing the form of the equations of motion. We, therefore, obtain:

$$\left[1 - 2(D-3)(D-4)\lambda(A')^2\exp(-2A)\right]\left(\partial_\sigma\partial^\sigma H_{\mu\nu} + \partial_\mu\partial_\nu H - \partial_\mu\partial^\sigma H_{\sigma\nu} - \partial_\nu\partial^\sigma H_{\sigma\mu} -\right.$$

$$\eta_{\mu\nu}\left[\partial_\sigma\partial^\sigma H - \partial^\sigma\partial^\rho H_{\sigma\rho}\right] + H''_{\mu\nu} - \eta_{\mu\nu}H'' + (D-2)A'\left[H'_{\mu\nu} - \eta_{\mu\nu}H'\right] +$$

$$\left\{\partial_\mu\partial_\nu\rho - \eta_{\mu\nu}\partial_\sigma\partial^\sigma\rho + \eta_{\mu\nu}\left[(D-2)A'\rho' + (D-1)(D-2)(A')^2\rho\right]\right\}\right) -$$

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$$4(D-4)\lambda \left[A'' - (A')^2\right] \exp(-2A)\Big(\partial_\sigma \partial^\sigma H_{\mu\nu} + \partial_\mu \partial_\nu H - \partial_\mu \partial^\sigma H_{\sigma\nu} - \partial_\nu \partial^\sigma H_{\sigma\mu} -$$
$$\eta_{\mu\nu}\left[\partial_\sigma \partial^\sigma H - \partial^\sigma \partial^\rho H_{\sigma\rho}\right] + (D-3)A'\left\{H'_{\mu\nu} - \eta_{\mu\nu}H'\right\}\Big) +$$
$$\eta_{\mu\nu}\rho\Big(2(D-2)\left[A'' - (A')^2\right]\left[1 - 4(D-3)(D-4)\lambda(A')^2 \exp(-2A)\right] +$$
$$\frac{4}{D-2}(\phi')^2\Big) = 0 \ , \tag{85}$$

$$\left[1 - 2(D-3)(D-4)\lambda(A')^2 \exp(-2A)\right]\Big(\left[\partial^\mu H_{\mu\nu} - \partial_\nu H\right]' + (D-2)A'\partial_\nu \rho\Big) = 0 \ , \tag{86}$$

$$\left[1 - 2(D-3)(D-4)\lambda(A')^2 \exp(-2A)\right]\Big(-\left[\partial^\mu \partial^\nu H_{\mu\nu} - \partial^\mu \partial_\mu H\right] + (D-2)A'H' -$$
$$(D-1)(D-2)(A')^2\rho\Big) + \frac{4}{D-2}(\phi')^2\rho = 0 \ , \tag{87}$$

$$\phi'\left[\rho' - H'\right] + \frac{D-2}{4}\rho V_\phi \exp(2A) = 0 \ . \tag{88}$$

Here we note that the graviscalar component cannot be gauged away after we perform the above gauge fixing.

The above equations further simplify in the smooth domain wall background discussed in the previous subsection. Thus, let

$$\chi(z) \equiv \alpha\zeta^2\sqrt{\lambda\kappa}\ y(z) \ , \tag{89}$$

where $y(z)$ can be computed using the map (59). We have:

$$\left[1 + \frac{4(D-4)}{D-2}\lambda\zeta^2\right](\partial_\sigma \partial^\sigma H_{\mu\nu} + \partial_\mu \partial_\nu H - \partial_\mu \partial^\sigma H_{\sigma\nu} - \partial_\nu \partial^\sigma H_{\sigma\mu} - \eta_{\mu\nu}\left[\partial_\sigma \partial^\sigma H - \partial^\sigma \partial^\rho H_{\sigma\rho}\right]) +$$
$$H''_{\mu\nu} - \eta_{\mu\nu}H'' + (D-2)A'\left[1 + \frac{4(D-3)(D-4)}{(D-2)^2}\lambda\zeta^2\right]\left[H'_{\mu\nu} - \eta_{\mu\nu}H'\right] +$$
$$\partial_\mu \partial_\nu \rho - \eta_{\mu\nu}\partial_\sigma \partial^\sigma \rho + \eta_{\mu\nu}(D-2)A'\rho' +$$
$$\eta_{\mu\nu}\rho\exp(2A)\left\{\zeta^2\left[2 - 3\cosh^{-2}(\chi)\right] + 2\Omega\tanh^2(\chi)\right\} = 0 \ , \tag{90}$$

$$\left[\partial^\mu H_{\mu\nu} - \partial_\nu H\right]' + (D-2)A'\partial_\nu \rho = 0 \ , \tag{91}$$

$$-\left[\partial^\mu \partial^\nu H_{\mu\nu} - \partial^\mu \partial_\mu H\right] + (D-2)A'H' + \rho\exp(2A)\left[\zeta^2\cosh^{-2}(\chi) - 2\Omega\tanh^2(\chi)\right] = 0 \ , \tag{92}$$

$$\rho' - H' + \frac{8(D-3)(D-4)}{D-2}\lambda\left[\zeta^2 + \Omega\right]A'\rho = 0 \ . \tag{93}$$

Here we note that not all of these equations are independent. Thus, differentiating (90) with ∂^μ, we obtain an equation which is identically satisfied once we take into account (91) as well as the on-shell expression for A. Also, if we take the trace of (90), then we obtain an equation which is identically satisfied once we take into account (91), (92) and (93) as well as the on-shell expression for A. This, as usual, is a consequence of Bianchi identities.

Now we are ready to discuss normalizable modes in the above domain wall background. Let us first consider the normalizable modes for the graviscalar ρ. Thus, we can eliminate $H_{\mu\nu}$ from (91), (92) and (93), which gives us the following second order equation for ρ:

$$\rho'' + \psi A'\rho' + \partial^\mu \partial_\mu \rho + F\rho = 0 \ , \tag{94}$$

where

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$$\psi \equiv D + \frac{8(D-3)(D-4)}{D-2}\lambda\zeta^2 \ , \tag{95}$$

and

$$F(z) \equiv 2(A')^2 \left[1 + \frac{2(D-3)(D-4)}{D-2}\lambda\zeta^2\right] + 2A'' \left[D - 2 + \frac{6(D-3)(D-4)}{D-2}\lambda\zeta^2\right] \ . \tag{96}$$

Let us now assume that ρ satisfies the $(D-1)$-dimensional Klein-Gordon equation

$$\partial^\mu\partial_\mu\rho = m^2\rho \ . \tag{97}$$

In the following we will assume that $m^2 \geq 0$. As to the $m^2 < 0$ modes, they cannot be normalizable - indeed, the domain wall is a kink-like object, and is therefore stable, so no tachyonic modes are normalizable.

We need to understand the asymptotic behavior of ρ at large z. To do this, it is convenient to rescale ρ as follows:

$$\rho \equiv \widetilde{\rho}\exp\left[-\frac{1}{2}\psi A\right] \ . \tag{98}$$

The equation (94) then reads:

$$\widetilde{\rho}'' + \left[m^2 + F - \frac{1}{2}\psi A'' - \frac{1}{4}\psi^2(A')^2\right]\widetilde{\rho} = 0 \ . \tag{99}$$

Note that at large z the functions F, A'' and $(A')^2$ go to zero as $\sim 1/z^2$. We therefore have the following leading behavior for $\widetilde{\rho}$ at large z:

$$\widetilde{\rho}(z) = C_1\cos(mz) + C_2\sin(mz) \ , \tag{100}$$

where C_1, C_2 are some constant coefficients.

Next, note that the norm for the graviscalar is given by

$$||\rho||^2 \propto \int dz \ \exp(DA)\rho^2 \ , \tag{101}$$

where the measure $\exp(DA)$ comes from $\sqrt{-G}$. In terms of $\widetilde{\rho}$ we have

$$||\rho||^2 \propto \int dz \ \exp\left[-\frac{8(D-3)(D-4)}{D-2}\lambda\zeta^2 A\right]\widetilde{\rho}^2 \ . \tag{102}$$

Since A goes to $-\infty$ at large z, we conclude that none of the $m^2 > 0$ modes are even plane-wave normalizable. Moreover, since the function F in (94) is non-trivial, we do not have a quadratically normalizable zero mode either. Thus, we conclude that ρ is *not* a propagating degree of freedom in the above background, and should be set to zero.

Next, let us turn to the normalizable modes for the graviton $H_{\mu\nu}$. From (91), (92) and (93) it follows that, since $\rho \equiv 0$, we have

$$\partial^\mu H'_{\mu\nu} = H' = 0 \ . \tag{103}$$

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This then implies that we can use the residual $(D-1)$-dimensional diffeomorphisms (for which $\omega \equiv 0$, and $\widetilde{\xi}_\mu$ are independent of z) to bring $H_{\mu\nu}$ into the transverse-traceless form:

$$\partial^\mu H_{\mu\nu} = H = 0 . \tag{104}$$

It then follows from (90) that for the modes of the form

$$H_{\mu\nu} = \xi_{\mu\nu}(x^\rho)\Sigma(z) , \tag{105}$$

where

$$\partial^\sigma \partial_\sigma \xi_{\mu\nu} = m^2 \xi_{\mu\nu} , \tag{106}$$

the z-dependent part of $H_{\mu\nu}$ satisfies the following equation:

$$\Sigma'' + \Psi_1 A'\Sigma' + \Psi_2^2 m^2 \Sigma = 0 , \tag{107}$$

where

$$\Psi_1 \equiv (D-2)\left[1 + \frac{4(D-3)(D-4)}{(D-2)^2}\lambda\zeta^2\right] , \tag{108}$$

$$\Psi_2 \equiv \left[1 + \frac{4(D-4)}{D-2}\lambda\zeta^2\right]^{\frac{1}{2}} . \tag{109}$$

Let us rescale Σ as follows:

$$\Sigma \equiv \widetilde{\Sigma}\exp\left[-\frac{1}{2}\Psi_1 A\right] . \tag{110}$$

The equation (107) now reads:

$$\widetilde{\Sigma}'' + \left[\Psi_2^2 m^2 - \frac{1}{2}\Psi_1 A'' - \frac{1}{4}\Psi_1^2 (A')^2\right]\widetilde{\Sigma} = 0 . \tag{111}$$

At large z we therefore have:

$$\widetilde{\Sigma}(z) = D_1 \cos\left(\Psi_2 m z\right) + D_2 \sin\left(\Psi_2 m z\right) , \tag{112}$$

where D_1, D_2 are some constant coefficients.

Next, note that the norm for the graviton is given by

$$||H_{\mu\nu}||^2 \propto \int dz \, \exp[(D-2)A]\Sigma^2 , \tag{113}$$

where, unlike the graviscalar case, the measure $\exp[(D-2)A]$ comes from $\sqrt{-G}R$. In terms of $\widetilde{\Sigma}$ we have

$$||H_{\mu\nu}||^2 \propto \int dz \, \exp\left[-\frac{4(D-3)(D-4)}{D-2}\lambda\zeta^2 A\right]\widetilde{\Sigma}^2 . \tag{114}$$

Thus, we see that none of the $m^2 > 0$ modes are even plane-wave normalizable. Unlike the graviscalar case, however, we do have a quadratically normalizable zero mode for $H_{\mu\nu}$. This zero mode is given by $\Sigma' = 0$.

Thus, as we see, in the above smooth domain wall background the only propagating degree of freedom is the zero mode of $H_{\mu\nu}$ corresponding to $(D-1)$-dimensional gravity. This result then makes it clear why in the $\zeta \to \infty$ limit, where we recover the aforementioned δ-function-like solitonic brane world solution, gravity is completely localized on the brane.

VI. CONJECTURE

In the previous sections we discussed a solitonic brane world solution with completely localized gravity arising in the Einstein-Hilbert-Gauss-Bonnet theory with a cosmological term. In $D = 5$ the Gauss-Bonnet combination is the only higher curvature term that we can consider in this context[9]. For higher D, however, we can add higher Euler invariants (*e.g.*, in $D = 7$ we can also include the 6-dimensional Euler invariant). In these cases we also expect to have similar solitonic brane world solutions.

Regardless of an explicit field theory realization of such a solitonic brane world, one question that immediately arises is whether we can have matter localized on the brane. If this brane is a D-brane-like object, then we could hope to have gauge and matter fields localized on the brane. In this context we would like to ask whether we can embed our scenario in the string theory framework. In fact, here we would like to take this a bit further and ask whether (at least in some cases) D-branes can be identified as such solitonic brane world solutions. In this section we will propose a conjecture (in a weak as well as strong form) according to which the answers to the above questions are positive. In the following we will focus on the case of 3-branes, although we expect that generalizations to other branes should also be possible.

A. D3-branes at Conical Singularities

In this subsection we will review some facts concerning parallel D3-branes near a conical singularity. Our discussion here will closely follow [29][10]. Thus, consider Type IIB in the presence of (large but finite number) N D3-branes at the conical singularity located at $r = 0$ in the 6-dimensional non-compact Ricci-flat manifold Y_6 with the metric

$$g_{IJ}dx^I dx^J = (dr)^2 + r^2 \gamma_{ij}dx^i dx^j \ . \tag{115}$$

Here γ_{ij} is a metric on a 5-dimensional manifold X_5, and the $r = 0$ point is a singularity unless X_5 is a 5-sphere. The condition that Y_6 is Ricci-flat implies that X_5 is an Einstein manifold of positive curvature. Note that in the special case where X_5 is a 5-sphere $\mathbf{S}^5$ the manifold Y_6 is actually flat.

In the above background the 10-dimensional metric has the following form ($\eta_{\mu\nu}$ is the flat 4-dimensional Minkowski metric):

$$ds^2 = Q^{-1/2}(r)\eta_{\mu\nu}dx^\mu dx^\nu + Q^{1/2}(r)\left[(dr)^2 + r^2\gamma_{ij}dx^i dx^j\right] \ , \tag{116}$$

where

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[9]Here the following remark is in order. The background in such warped compactifications is not affected by higher curvature terms constructed solely from the Weyl tensor (which is the "traceless" part of the Riemann tensor) as such terms vanish in conformally flat backgrounds. Note, however, that such terms would affect the graviton propagator in the bulk.

[10]This setup was originally discussed in [30].

$$Q(r) \equiv 1 + \frac{L^4}{r^4} , \quad L^4 \equiv 4\pi g_s N(\alpha')^2 . \tag{117}$$

The near horizon limit of the above geometry, that is, the limit $r \to 0$, coincides with that of $\text{AdS}_5 \times X_5$. According to the arguments in [31], we expect that the field theory on the D3-branes at the above conical singularity should be dual to Type IIB string theory on $\text{AdS}_5 \times X_5$ (with N units of 5-form flux on X_5).

Let us now discuss the field theory on the branes. In the special case where $X_5 = \mathbf{S}^5$ the manifold Y_6 is smooth (it is simply $\mathbf{R}^6$). The gauge theory on the branes is then $\mathcal{N} = 4$ $U(N)$ SYM theory, which is conformal. This fact is consistent with the conformal property of AdS_5 and the aforementioned duality between the gauge theory on the branes and Type IIB on $\text{AdS}_5 \times X_5$ [32,33].

Let us now consider cases where X_5 is *not* a 5-sphere (but is still a compact Einstein manifold). In this case supersymmetry is at least partially broken. The simplest examples of such manifolds are orbifolds of $\mathbf{S}^5$: $X_5 = \mathbf{S}^5/\Gamma$ [34], where Γ is a finite discrete subgroup of $SO(6)$, or, more precisely, of $\text{Spin}(6)$ as we are dealing with a theory containing fermions. Note that the latter group is the R-parity group of $\mathcal{N} = 4$ SYM. The gauge theories on branes at the corresponding orbifold singularities were discussed in detail in [35,36][11]. Thus, if $\Gamma \subset SU(3)(SU(2))$, then the corresponding gauge theory is $\mathcal{N} = 1$ ($\mathcal{N} = 2$) supersymmetric. Otherwise, supersymmetry is completely broken. Here we would like to consider some simple examples of such theories which capture the main point we would like to make.

Thus, let us consider the simplest example of such a theory. Let $\Gamma = \mathbf{Z}_2$, whose generator R has the following action on the complex coordinates z_1, z_2, z_3 on the 5-sphere (the 5-sphere is given by $|z_1|^2 + |z_2|^2 + |z_3|^2 = \rho^2$, where ρ is its radius): $Rz_{1,2} = -z_{1,2}$, $Rz_3 = z_3$. The gauge theory on the branes is given by the $\mathcal{N} = 2$ $U(n) \otimes U(n)$ gauge theory with 2 copies of hypermultiplets in $(\mathbf{n}, \overline{\mathbf{n}})(+1, -1)$ and $(\overline{\mathbf{n}}, \mathbf{n})(-1, +1)$, where the $U(1)$ charges are given in parenthesis. Note that nothing is charged under the diagonal $U(1)_S$ (which corresponds to the brane center-of-mass degree of freedom), but the matter fields are charged under the anti-diagonal $U(1)_A$. (The generators of these $U(1)$'s are given by $Q_S = \frac{1}{\sqrt{2}}(Q_1 + Q_2)$ and $Q_A = \frac{1}{\sqrt{2}}(Q_1 - Q_2)$, where Q_1, Q_2 are the generators of the original $U(1)$'s.) This implies that $U(1)_A$ runs, and decouples in the infra-red. On the other hand, the non-Abelian part of the gauge group is conformal.

Let us briefly discuss another example of this type, which has $\mathcal{N} = 1$ supersymmetry. Let the orbifold group be $\Gamma = \mathbf{Z}_3$, whose generator θ has the following action: $\theta z_{1,2,3} = \exp(2\pi i/3)z_{1,2,3}$. Then the gauge theory is the $\mathcal{N} = 1$ $U(n) \otimes U(n) \otimes U(n)$ gauge theory with 3 copies of chiral multiplets in $(\mathbf{n}, \overline{\mathbf{n}}, \mathbf{1})(+1, -1, 0)$, $(\mathbf{1}, \mathbf{n}, \overline{\mathbf{n}})(0, +1, -1)$ and $(\overline{\mathbf{n}}, \mathbf{1}, \mathbf{n})(-1, 0, +1,)$, call them A_a, B_a, C_a, $a = 1, 2, 3$, and the renormalizable superpotential

$$\mathcal{W} = \epsilon_{abc} A_a B_b C_c . \tag{118}$$

Once again, there is nothing charged under the diagonal $U(1)$, while the matter fields carry

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[11]The generalization to gauge theories on branes at orientifold singularities was subsequently discussed in [37].

non-trivial charges under the other two linear combinations. These two $U(1)$'s decouple in the infra-red, and the non-Abelian part of the gauge group is conformal.

The point that we would like to make here, however, is that the gauge theories in the above examples as well as for *any* other non-trivial choice of the orbifold group Γ are conformal only in the infra-red. In particular, in the orbifold examples we always have running $U(1)$'s under which matter fields are charged. This implies that the brane world-volume theory is actually not conformal, in particular, in the ultra-violet we have non-trivial loop corrections.

Before we turn to the upshot of the above discussion, let us give one more example, where X_5 is not an orbifold of $\mathbf{S}^5$. This example is instructive as already the non-Abelian part of the corresponding gauge theory is not conformal in the ultra-violet. Thus, consider the case where $X_5 = T^{1,1} = (SU(2) \times SU(2))/U(1)$ [29]. The gauge theory then is given by the $\mathcal{N} = 1$ $U(n) \otimes U(n)$ gauge theory with 2 copies of chiral multiplets in $(\mathbf{n}, \overline{\mathbf{n}})(+1, -1)$ and $(\overline{\mathbf{n}}, \mathbf{n})(-1, +1)$, call them A_a, B_a, $a = 1, 2$, and the non-renormalizable superpotential

$$\mathcal{W} = \epsilon_{ab}\epsilon_{cd}A_a B_c A_b B_d \ . \tag{119}$$

As in the $X_5 = \mathbf{S}^5/\mathbf{Z}_2$ example[12], here we also have a running (anti-diagonal) $U(1)$. Note, however, that already the non-Abelian part of the gauge group is *not* conformal in the ultra-violet. As to the infra-red, the anti-diagonal $U(1)$ decouples (the diagonal $U(1)$ is free to begin with), and the non-Abelian part of the theory flows into a conformal fixed point.

B. Gravity on D-branes

Thus, as we see, as long as X_5 is not a 5-sphere, the gauge theory is not conformal in the ultra-violet, albeit it is conformal in the infra-red[13]. We therefore have non-trivial loop corrections in the ultra-violet. This has important consequences. Thus, as was pointed out in [15,16], if we have non-conformal theory on a brane, loop corrections are generically expected to generate graviton propagator on the brane. That is, at the quantum level we expect (among other terms) the Einstein-Hilbert term to be generated in the world-volume of D3-branes with non-conformal theories. The question we would like to ask here is whether we can understand the presence of this term in the world-volume theory of D3-branes in the dual Type IIB picture. Here we would like to make a conjecture which implies that the answer to this question is positive.

Weak Form of the Conjecture

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[12]In fact, the $X_5 = \mathbf{S}^5/\mathbf{Z}_2$ and $X_5 = T^{1,1}$ cases are related as follows [29]. There is a fixed circle in $\mathbf{S}^5/\mathbf{Z}_2$. Blowing up this fixed circle breaks half of the supersymmetries, and deforms the $\mathcal{N} = 2$ gauge theory corresponding to the $X_5 = \mathbf{S}^5/\mathbf{Z}_2$ case to the $\mathcal{N} = 1$ gauge theory corresponding to the $X_5 = T^{1,1}$ case.

[13]If we include orientifold planes [37], then one can construct examples where the gauge theory (at least in the $\mathcal{N} = 1$ cases) is not conformal even in the infra-red. In these cases, however, some caution is needed due to subtleties discussed in [38,39].

From the above discussion it is clear that the gauge theory on D3-branes at the conical singularity in Y_6 can be dual to Type IIB on $AdS_5 \times X_5$ only in the infra-red limit of the gauge theory unless $X_5 = \mathbf{S}_5$. It is precisely in the latter case that Y_6 is actually non-singular (and, moreover, flat), and we have maximal supersymmetry. In other cases supersymmetry is at least partially broken. We can therefore expect that the $AdS_5 \times X_5$ background is corrected by higher curvature (or, more generally, higher derivative) terms[14]. That is, here we propose that, if $X_5 \neq \mathbf{S}_5$, $AdS_5 \times X_5$ with N units of 5-form flux is *not* an exact background of Type IIB. Moreover, we propose that we have *gravity* on D3-branes, that is, we have the Einstein-Hilbert term in their world-volume action. In the gauge theory language this term arises from loop corrections due to non-conformal matter fields[15]. Here we conjecture that there is a dual Type IIB description, which is valid beyond the infra-red limit of the gauge theory, and in this description the effects corresponding to having gravity in the D-brane world-volume are due to higher curvature terms (which are responsible for the fact that $AdS_5 \times X_5$ is not an exact background). Here we note that, if such a duality indeed holds, the fact that $AdS_5 \times X_5$ cannot be an exact background is evident from the fact that the dual gauge theory is not conformal in the ultra-violet. On the other hand, the origin of the aforementioned higher curvature terms *a priori* might be less evident. Here we would like to stress that these are *not* the higher curvature terms (such as the (Weyl)^4 terms) that are already present in a flat 10-dimensional background of Type IIB [42]. Rather, these higher curvature terms are intrinsically due to the compactification. Thus, for instance, let us consider the orbifold cases $X_5 = \mathbf{S}^5/\Gamma$. In these cases we have twisted sector fields (including those that are massive) which contribute into the higher curvature terms, and these terms are intrinsically "5-dimensional". Note that the appearance of these intrinsically "5-dimensional" higher curvature terms goes hand-by-hand with (partial) supersymmetry breaking.

Strong Form of the Conjecture

The above (form of the) conjecture is somewhat vague in the sense that it does not specify how the $AdS_5 \times X_5$ background is modified due to the aforementioned higher curvature terms. Clearly, this is not a simple question, but, nonetheless, here we would like to make a guess based on the following observations. First, in the cases where the D-brane world-volume gauge theory is conformal in the infra-red, we expect that the dual Type IIB background should be at least asymptotically $AdS_5 \times X_5$. Second, we have gravity on D-branes. Third, at least in some limit D-branes are δ-function-like objects. Here we conjecture that if we take into account higher curvature terms in Type IIB compactification on X_5 with N units of 5-form flux on X_5, (at least in some cases) in the 5-dimensional space transverse to X_5

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[14]As was argued in [40,41], the $AdS_5 \times \mathbf{S}_5$ background is an exact solution of Type IIB, which is due to the fact that we have maximal supersymmetry in this case.

[15]In the $X_5 = \mathbf{S}^5/\Gamma$ orbifold cases, where non-conformality is due to running $U(1)$'s, the Einstein-Hilbert term is expected to arise at the two-loop order. In the cases where already the non-Abelian part of the gauge group is non-conformal, this is expected to occur at the one-loop order. This can be seen by using arguments similar to those in [36].

we have *solitonic* solutions, which (at least in a certain limit) are δ-function like, and couple to the Ramond-Ramond 4-form. We propose that these solitonic solutions are nothing but D3-branes. Moreover, there is gravity localized on these solitonic D3-branes, while the 5-dimensional space transverse to X_5 is asymptotically AdS_5. Since here we are talking about solitonic δ-function-like branes, in the light of our discussions in the previous sections here we also propose that gravity is *completely* localized[16] on these branes, that is, it does not propagate in the 5th dimension (albeit there are heavy Kaluza-Klein modes propagating in the other 5 directions along X_5, which is compact)[17].

One of the implications of this conjecture would be that the loop corrections, apart from those due to the heavy KK/string thresholds, are 4-dimensional and not 5-dimensional[18], just as in the solitonic brane world solution we discussed in this paper. Moreover, in this picture D-branes are *non-singular* solitonic solutions of full Type IIB string theory[19]. Finally, this conjecture relates observations of this paper, the scenarios discussed in [15,16], and string theory in the spirit of [31], which might at first seem unrelated.

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[16]The reason why we expect gravity to be completely localized on such branes is due to the fact that the latter are δ-function-like solitonic solutions which do not break diffeomorphisms explicitly but spontaneously. This then implies that the graviscalar is a gauge degree of freedom. On the other hand, if gravity is not completely localized, that is, if we have massive Kaluza-Klein modes propagating in the bulk as in, say, [14], then at short distances (that is, large momenta) gravity is expected to become 5-dimensional, which requires that the coupling of the graviscalar to the *non-conformal* brane matter be non-vanishing [22], but this is not possible if we can gauge the graviscalar away. A more detailed discussion of this point will be given in [24].

[17]Here we would like to make the following remark. In the above picture, in the Type IIB language the analog of the "fine-tuning" relation (6) should arise as a result of the fact that the relevant higher curvature terms come from the compactification on X_5, so that the corresponding couplings should be determined by the geometry of X_5. On the other hand, the volume of X_5 as well as the bulk vacuum energy density are expected to be related to the 5-form flux, which is quantized.

[18]The theory is in some vague sense "holographic". At present it is unclear whether this can in any way be tied to the observations of [43,44,15,45,28,16] that bulk supersymmetry might in some cases control the brane cosmological constant.

[19]Note that according to the stronger form of the above conjecture, in the corresponding cases we do not really have two (gauge theory *vs.* Type IIB) descriptions. Rather, D-branes are part of the Type IIB background. Note that $X_5 = \mathbf{S}^5$ case (and, perhaps, some other cases) is exceptional in this sense. Nonetheless, perhaps this case can be thought of as some limit of the generic situation described in the above conjecture.

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